Int. J. Solids Structures Vol. 16, pp. 387–396
Pergamon Press Ltd., 1980. Printed in Great Britain

A TENSIONLESS CONTACT WITHOUT FRICTION BETWEEN AN ELASTIC LAYER AND AN ELASTIC FOUNDATION†

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(*Received 8 January 1979; in revised form 27 August 1979*)

Abstract—The plane contact problem for an infinite elastic layer lying on an elastic half space is considered. The layer is acted upon by a uniform clamping pressure p_0, a uniform vertical body force $\rho_1 g$ due to the effect of gravity in the layer and a concentrated vertical line load P. It is assumed that the contact between the layer and the half space is frictionless and that only compressive normal tractions can be transmitted through the interface. The contact along the interface will be continuous if the value of P is less than a critical value P_{cr}. However, for $P > P_{cr}$ interface separation takes place along a certain finite region. First, the problem of continuous contact is solved and the value P_{cr} is determined. Then the discontinuous contact problem is formulated in terms of a singular integral equation. Two loading conditions are considered assuming that the concentrated line load P is either a tensile load or a compressive load. Numerical results for P_{cr}, contact stress distributions, and separation regions are given for various material combinations.

1. INTRODUCTION

This paper is concerned with the plane contact problem for an elastic layer resting on an elastic foundation. The problem has attracted considerable attention in the past due to its applicability to a variety of important problems related to foundation-superstructure interaction (see, e.g. [1–7]). In most of previous studies the layer is pressed locally against the foundation. In the earlier studies the contact between the layer and the foundation was assumed to be either perfect adhesion or frictionless and to be continuous. However, in [1] it was shown that due to the bending of the layer under local compressive loads, in the absence of gravity effects, the contact area would decrease to a finite size which is independent of the magnitude of the load. This property holds also for loading through a flat-ended rigid stamp with sharp edges whereas for other stamp profiles the size of the contact area is a function of the resultant compressive force [8, 9]. In most of previous publications the effect of gravity is neglected. Under tensile loads one cannot obtain realistic solutions for frictionless horizontal layer problems without considering the effect of gravity or a possible clamping pressure. Some examples taking the effect of gravity into account may be found in [10–14]. In these references the layer rests on a frictionless, horizontal, rigid foundation. Studies [10–13] consider plane problems whereas [14] considers the axisymmetric problem. In such problems the contact area is infinite and when the magnitude of the external load exceeds a certain critical value a separation takes place between the layer and the foundation.

In this paper, the plane elastostatic problem of an infinite horizontal layer lying on a half space is considered. It is assumed that the frictionless layer-subspace interface can transmit compressive normal tractions only. The layer is under the action of a uniform clamping pressure on its top surface and a uniform body force due to gravity. In addition, a concentrated vertical line load is applied on the top surface which can be tensile or compressive. The problem has been solved in [11 and 12] for a rigid subspace and for tensile and compressive line loads, respectively. On the other hand in most structural applications the foundation is elastic with a Young's modulus which is generally less than that of the layer. Hence, for such problems the results given in this paper are somewhat more realistic.

2. FORMULATION

Consider an infinite elastic layer of thickness h in smooth contact with a semi-infinite elastic foundation. The geometry and coordinate system are shown in Fig. 1. Let $\rho_1 g$ be the body force

†This work was partially supported by NSF under the Grant ENG-78-09737.
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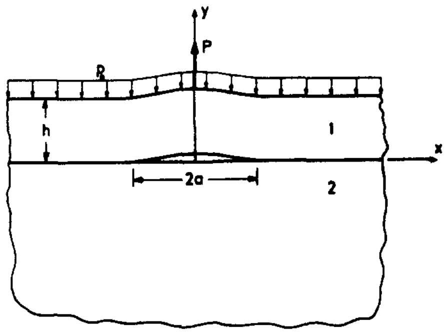

Fig. 1. Geometry for the elastic layer lying on an elastic foundation (tensile line load, discontinuous contact case).

density acting vertically in the layer and note that the body force acting in the foundation is neglected since it does not disturb the contact pressure distribution. Writing

$$u_i = u_{ip} + u_{ih}, \quad v_i = v_{ip} + v_{ih}, \quad (i = 1, 2), \tag{1a, b}$$

the particular part of the displacement components corresponding to $\rho_1 g$ and the clamping pressure p_0 may be obtained separately as[11]

$$u_{1p} = \frac{3-\kappa_1}{16\mu_1} p_e x, \tag{2a, b}$$

$$v_{1p} = \frac{1-\kappa_1}{1+\kappa_1}\frac{\rho_1 g}{2\mu_1} y(h-y) - \frac{1+\kappa_1}{16\mu_1} p_e h y + A,$$

$$u_{2p} = \frac{3-\kappa_2}{8\mu_2} p_e x, \tag{3a, b}$$

$$v_{2p} = -\frac{1+\kappa_2}{8\mu_2} p_e y + A,$$

where

$$p_e = \rho_1 g h + p_0, \tag{4}$$

u and v are the x and y-components of the displacement vector, μ is the shear modulus, $\kappa = 3 - 4\nu$ for plane strain, $\kappa = (3-\nu)/(1+\nu)$ for plane stress, ν being the Poisson's ratio. The subscripts 1 and 2 refer to the layer and the foundation, respectively. The constant A appearing in (2b) and (3b) is the (yet unknown) rigid body displacement term.

Now, observing that $x = 0$ is a plane of symmetry, the homogeneous part of the displacement components for the layer and for the half space may be written as[8]

$$u_{1h} = \frac{2}{\pi}\int_0^\infty [(B + yC)\,e^{-sy} + (D + yE)\,e^{sy}]\sin(sx)\,ds,$$

$$v_{1h} = \frac{2}{\pi}\int_0^\infty \left\{\left[B + \left(\frac{\kappa_1}{s} + y\right)C\right]e^{-sy} + \left[-D + \left(\frac{\kappa_1}{s} - y\right)E\right]e^{sy}\right\}\cos(sx)\,ds, \tag{5a, b}$$

$$u_{2h} = \frac{2}{\pi}\int_0^\infty (F + yG)\,e^{sy}\sin(sx)\,ds,$$

$$v_{2h} = \frac{2}{\pi}\int_0^\infty \left[-F + \left(\frac{\kappa_2}{s} - y\right)G\right]e^{sy}\cos(sx)\,ds. \tag{6a, b}$$

The stress components of interest are found from (1) to (3), (5) and (6) using Hooke's law as follows:

$$\sigma_{yy1}=\frac{4\mu_1}{\pi}\int_0^\infty\left\{-\left[s(B+yC)+\frac{\kappa_1+1}{2}C\right]e^{-sy}\right.$$

$$\left.+\left[-s(D+yE)+\frac{\kappa_1+1}{2}E\right]e^{sy}\right\}\cos(sx)\,ds+\rho_1 gy-p_e, \tag{7a, b}$$

$$\tau_{xy1}=\frac{4\mu_1}{\pi}\int_0^\infty\left\{-\left[s(B+yC)+\frac{\kappa_1-1}{2}C\right]e^{-sy}\right.$$

$$\left.+\left[s(D+yE)-\frac{\kappa_1-1}{2}E\right]e^{sy}\right\}\sin(sx)\,ds,$$

$$\sigma_{yy2}=\frac{4\mu_2}{\pi}\int_0^\infty\left[-s(F+yG)+\frac{\kappa_2+1}{2}G\right]e^{sy}\cos(sx)\,ds-p_e,$$

$$\tau_{xy2}=\frac{4\mu_2}{\pi}\int_0^\infty\left[s(F+yG)-\frac{\kappa_2-1}{2}G\right]e^{sy}\sin(sx)\,ds. \tag{8a, b}$$

The unknown functions $B,\dots,G$ are determined by using the continuity and boundary conditions at $y=0$ and $y=h$.

3. THE CASE OF CONTINUOUS CONTACT ($0<P<P_{cr}$)

Let the layer be subjected to a uniform pressure p_0 and a concentrated lifting force P (per unit length in z-direction) along its boundary $y=h$. If P is sufficiently small the contact along the interface $y=0$ will be continuous and $B,\dots,G$ should be determined from the following boundary and continuity conditions:

$$\tau_{xy1}(x,h)=0,\quad \sigma_{yy1}(x,h)=\frac{P}{2}\delta(x)-p_0,\quad (0\le x<\infty), \tag{9a, b}$$

$$\tau_{xy1}(x,0)=0,\quad \tau_{xy2}(x,0)=0,\quad 0\le x<\infty,$$

$$\sigma_{yy1}(x,0)=\sigma_{yy2}(x,0),\quad 0\le x<\infty,$$

$$\frac{\partial}{\partial x}[v_1(x,+0)-v_2(x,-0)]=0,\quad 0\le x<\infty \tag{10a–d}$$

where, aside from a rigid body displacement, (10d) is equivalent to $v_1(x,0)=v_2(x,0)$. One should note that determination of the rigid body displacement term A requires an additional condition in the form of prescribing the vertical displacement of an arbitrary point. After determining $B,\dots,G$ from (9) and (10) the normalized contact pressure becomes

$$p(x)=1-\lambda\frac{2}{\pi}\int_0^\infty\frac{1}{\Delta}[(\omega+1)e^{\omega}+(\omega-1)e^{-\omega}]\cos\left(\frac{\omega x}{h}\right)d\omega,\quad (0\le x<\infty), \tag{11}$$

where

$$p(x)=-\sigma_{yy1}(x,0)/p_e,\quad \lambda=P/p_e h,$$

$$\Delta=(1+m)e^{2\omega}+4\omega-2m(2\omega^2+1)-(1-m)e^{-2\omega},$$

$$\omega=sh,\quad m=\mu_1(\kappa_2+1)/\mu_2(\kappa_1+1). \tag{12a–e}$$

From (11) it is seen that $p(x)=1$ for $\lambda=0$ and up to a certain value of λ $p(x)$ remains positive and the contact on $y=0$ plane remains continuous. The critical value of the load factor $\lambda=\lambda_{cr}$ at which the interface separation starts at $x=0$ can be obtained from (11) by using the condition

$$p(0)=0, \tag{13}$$

giving

$$\frac{1}{\lambda_{cr}}=\frac{2}{\pi}\int_0^\infty\frac{1}{\Delta}[(\omega+1)e^{\omega}+(\omega-1)e^{-\omega}]\,d\omega. \tag{14}$$

4. THE CASE OF DISCONTINUOUS CONTACT ($P>P_{cr}$)

When the value of P exceeds $P_{cr}=\lambda_{cr}p_e h$ the interface separation takes place around $x=0$ as shown in Fig. 1. Equations (1)–(8) and the conditions (9), (10a–c) are still valid. However, (10d) should be replaced by the following mixed boundary conditions

$$\sigma_{yy}(x,0)=0,\quad (0\leq x<a),$$

$$\frac{\partial}{\partial x}[v_1(x,+0)-v_2(x,-0)]=0,\quad (a<x<\infty), \tag{15a, b}$$

where $2a$ is the length of the separation region. Defining the following function

$$f(x)=\frac{\partial}{\partial x}[v_1(x,+0)-v_2(x,-0)],\quad (0\leq x<\infty), \tag{16}$$

$B,\dots,G$ can be determined in terms of $f(x)$ from (9), (10a–c), and (16). Then (15a) gives the following singular integral equation

$$\int_{-1}^{1}\left[\frac{1}{t-r}+k_1(r,t)\right]g(t)\,\mathrm{d}t+\lambda k_2(r)=\pi,\quad (-1<r<1), \tag{17}$$

where

$$g(t)=\frac{4\mu_1}{1+\kappa_1}\frac{f(at)}{(1+m)p_e},$$

$$k_1(r,t)=-2\left(\frac{a}{h}\right)\int_0^{\infty}\frac{1}{\Delta}(1+2\omega+2\omega^2-\mathrm{e}^{-2\omega})\sin\left[\frac{a}{h}\omega(t-r)\right]\mathrm{d}\omega,$$

$$k_2(r)=2\int_0^{\infty}\frac{1}{\Delta}[(\omega+1)\,\mathrm{e}^{\omega}+(\omega-1)\,\mathrm{e}^{-\omega}]\cos\left(\frac{a}{h}\omega r\right)\mathrm{d}\omega,$$

$$r=x/a. \tag{18a–d}$$

Equation (15b) is seen to be equivalent to

$$g(t)=0,\quad (1<t<\infty),\qquad \int_{-1}^{1}g(t)\,\mathrm{d}t=0. \tag{19a, b}$$

Because of symmetry $g(t)=-g(-t)$. Therefore, the single-valuedness condition (19b) is automatically satisfied. Because of the requirement of smooth contact at $x=a$, $f(a)=0$ and the index of the singular integral equation (17) is -1. Consequently the function $g(t)$ may be expressed in the form

$$g(t)=\phi(t)(1-t^2)^{1/2},\qquad (-1<t<1), \tag{20}$$

where $\phi(t)$ is a bounded odd function in $[-1,1]$. The solution $g(t)$ must satisfy the following consistency condition[15]

$$\int_{-1}^{1}\frac{\mathrm{d}t}{(1-t^2)^{1/2}}\left[1-\frac{\lambda}{\pi}k_2(t)-\frac{1}{\pi}\int_{-1}^{1}k_1(t,y)g(y)\,\mathrm{d}y\right]=0. \tag{21}$$

from which the length of the separation region is determined.

The normalized contact pressure now becomes

$$p(x)=p(ar)=1-\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{t-r}+k_1(r,t)\right]g(t)\,\mathrm{d}t-\frac{\lambda}{\pi}k_2(r),\quad (r>1). \tag{22}$$

The singular integral equation (17) can easily be reduced to the following system of linear algebraic equations by employing the appropriate Gauss–Chebyshev integration formula[16]

$$\sum_{i=1}^{n} \frac{1-t_i^2}{n+1}\left[\frac{1}{t_i-r_j}+k_1(r_j,t_i)\right]\phi(t_i)+\frac{\lambda}{\pi}k_2(r_j)=1,$$
$$(j=1,\ldots,n+1), \tag{23}$$

where

$$t_i=\cos\left(\frac{i\pi}{n+1}\right),\quad (i=1,\ldots,n),$$
$$r_j=\cos\left(\frac{2j-1}{n+1}\frac{\pi}{2}\right),\quad (j=1,\ldots,n+1). \tag{24a, b}$$

Note that (23) contains $(n+1)$ equations to determine $(n+1)$ unknowns $\phi(t_i)$, $(i=1,\ldots,n)$, and a. It can be shown that by using this technique to solve the integral equation the consistency condition (21) is automatically satisfied[16].

5. THE CASE OF COMPRESSIVE FORCE ($P<0$)

Now suppose that the concentrated lifting force P in Fig. 1 is replaced by a compressive force (see the insert in Figs. 8–11). The formulation given in Section 2 is still valid. If the value of compressive line load P is sufficiently small, the contact along the interface will be continuous and eqns (9a) and (10) will still be valid. However, (9b) should be replaced by

$$\sigma_{yy1}(x,h)=-\frac{P}{2}\delta(x)-p_0, \tag{25}$$

and hence the normalized contact pressure defined by (12a) becomes

$$p(x)=1+\lambda\frac{2}{\pi}\int_0^\infty \frac{1}{\Delta}[(\omega+1)\,e^{\omega}+(\omega-1)\,e^{-\omega}]\cos\left(\frac{\omega x}{h}\right)d\omega,\quad (0\leq x<\infty). \tag{26}$$

This expression is applicable for $\lambda<\lambda_{cr}$ for which $p(x)>0$. When $\lambda=\lambda_{cr}$, $p(x)$ becomes zero and interface separation starts at some location $x=x_{cr}\neq 0$. The critical load factor λ_{cr} and the corresponding location of initiation of interface separation x_{cr} can be determined through the use of following conditions

$$p(x_{cr})=0,\quad \frac{d}{dx}p(x_{cr})=0. \tag{27a, b}$$

For $\lambda>\lambda_{cr}$ the pressure will be zero in a certain range $b<|x|<c$ (see the insert in Fig. 11). In this case the problem may be formulated as

$$\int_{-1}^{1}\left[\frac{1}{t-r}+\frac{1}{t+r+2\frac{c+b}{c-b}}+k_1(r,t)\right]g(t)\,dt-\lambda k_2(r)=\pi,$$
$$(-1<r<1), \tag{28}$$

where now

$$g(t)=\frac{4\mu_1}{1+\kappa_1}\frac{1}{(1+m)p_e}f\left(\frac{c-b}{2}t+\frac{c+b}{2}\right),$$

$$k_1(r,t)=-\left(\frac{c-b}{h}\right)\int_0^\infty \frac{1}{\Delta}(1+2\omega+2\omega^2-e^{-2\omega})\left\{\sin\left[\omega\frac{c-b}{2h}(t-r)\right]\right.$$
$$\left.+\sin\omega\left[\frac{c-b}{2h}(t+r)+\frac{c+b}{h}\right]\right\}d\omega,$$

$$k_2(r) = 2\int_0^{\infty} \frac{1}{\Delta}[(\omega+1)e^{\omega} + (\omega-1)e^{-\omega}]\cos\left[\omega\left(\frac{c-b}{2h}r + \frac{c+b}{2h}\right)\right]d\omega,$$

$$r = \frac{2x}{c-b} - \frac{c+b}{c-b}. \tag{29a–d}$$

The singular integral equation (28) would then reduce to the following system of equations

$$\sum_{i=1}^{n} \frac{1-t_i^2}{n+1}\left[\frac{1}{t_i - r_j} + \frac{1}{t_i + r_j + 2\dfrac{c+b}{c-b}} + k_1(r_j, t_i)\right]\phi(t_i) - \frac{\lambda}{\pi}k_2(r_j) = 1,$$

$$(j = 1, \ldots, n+1), \tag{30}$$

where $\phi(t)$ and t_i, r_j are defined in (20) and (24), respectively. In this case the single-valuedness condition (19b) may be expressed as [16]

$$\sum_{i=1}^{n} \frac{1-t_i^2}{n+1}\phi(t_i) = 0. \tag{31}$$

Equations (30) and (31) constitute a system of $(n+2)$ equations for $(n+2)$ unknowns, $\phi(t_i)$, $(i = 1, \ldots, n)$, b and c. Again the consistency condition is satisfied automatically [16]. The normalized contact pressure will in this case be

$$p(x) = 1 - \frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{t-r} + \frac{1}{t + r + 2\dfrac{c+b}{c-b}} + k_1(r,t)\right]g(t)\,dt + \frac{\lambda}{\pi}k_2(r),$$

$$(|r| > 1). \tag{32}$$

6. RESULTS

Referring to eqn (12) for the definitions of the dimensionless bielastic constant m and the dimensionless load factor λ, Fig. 2 shows the relation between m and the critical load factor λ_{cr} at which interface separation initiates. λ_{cr1} and λ_{cr2} are the critical load factors corresponding to tensile and compressive line loads, respectively. $m = 0$ represents the case of an elastic layer resting on a rigid foundation [11, 12] whereas $m = 1$ may represent a layer and a foundation of identical materials. For $m = 0$, $\lambda_{cr1} = 1.088$ and $\lambda_{cr2} = 44.139$ are obtained which are the values given in [11, 12]. Results of these references were checked to be in good agreement with the present analysis. λ_{cr} increases with increasing m. This is expected due to the fact that as m increases the layer gets stiffer relative to the foundation and it becomes harder to bend the layer.

Figure 3 shows the relationship between the tensile load factor and the length of the separation region. For a fixed separation area the stiffer layer requires a greater load factor. For a fixed value of m separation region gets larger as λ increases. Figures 4–7 show the normalized contact pressure distributions for various values of m and λ for tensile line load case. The contact pressure is zero at $x = 0$ for $\lambda = \lambda_{cr}$. It tends to unity as x/h increases and has peaks around $x = \pm a$. The lifted portion of the layer is partially supported by these "concentrated" pressure peaks around $x = \pm a$. Consider, for example, the case of $m = 0.1$ and $a = 2h$ for which $\lambda = 2.84$ (see Fig. 3). In this case the lifting tensile load (per unit length in z-direction) is $P = 2.84p_e h$ whereas the downward force acting on the lifted portion of the layer is $2ap_e = 4p_e h$. The difference $1.16p_e h$ is supported by the peaks of the pressure distribution. Contact pressure distribution is heavily dependent on m and exhibits smoothness as m increases. For sufficiently large values of m the layer, which will be considerably stiffer than the foundation, is lifted as a whole instead of being bent and therefore pressure concentrations would be insignificant.

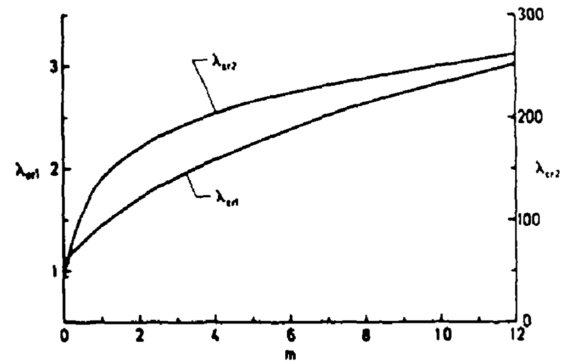

Fig. 2. Variation of the critical load factor $\lambda_{cr} = P_{cr}/(\rho_1 g h + p_0)h$ with $m = \mu_1(\kappa_2 + 1)/\mu_2(\kappa_1 + 1)$. ($\lambda_{cr1}$: tensile line load, λ_{cr2}: compressive line load).

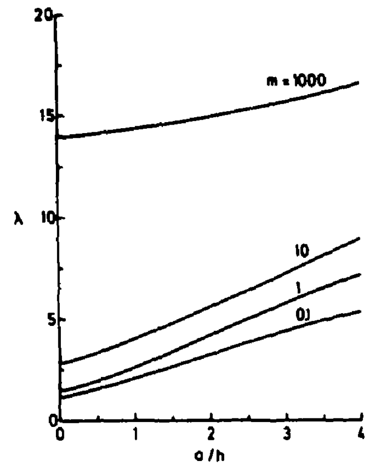

Fig. 3. The relation between the load factor λ and the half length of the separation region for tensile line load case.

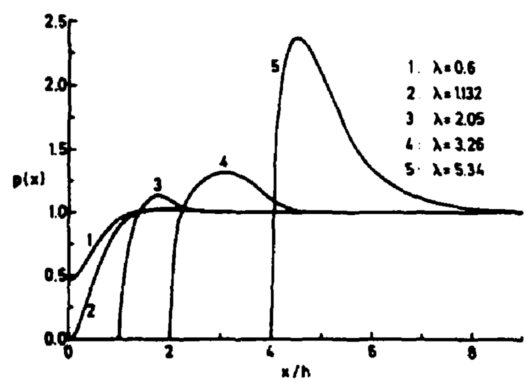

Fig. 4. Normalized contact pressure distributions for $m = 0.1$ (tensile line load case).

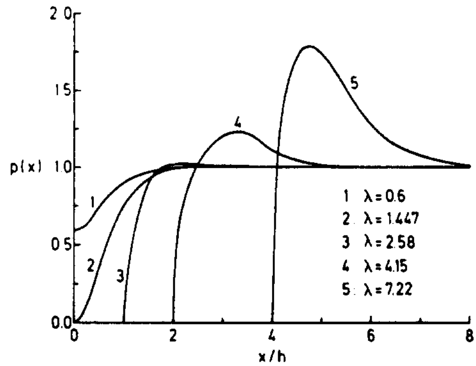

Fig. 5. Normalized contact pressure distributions for $m = 1$ (tensile line load case).

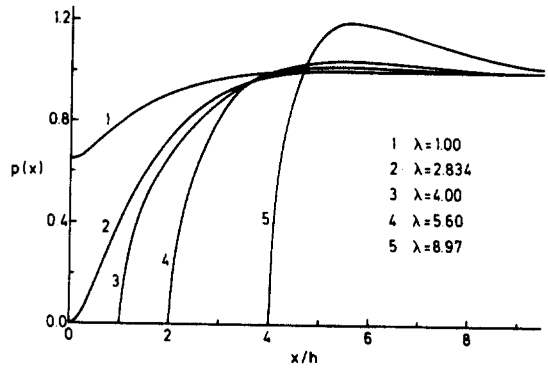

Fig. 6. Normalized contact pressure distributions for $m = 10$ (tensile line load case).

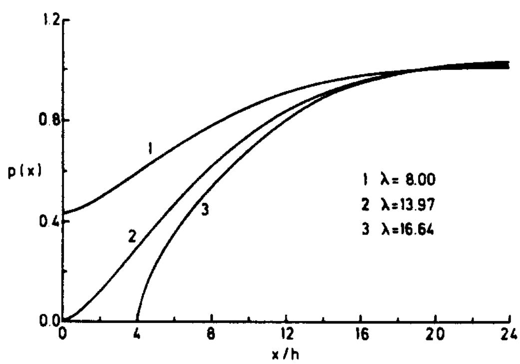

Fig. 7. Normalized contact pressure distributions for $m = 1000$ (tensile line load case).

Figures 8–10 show the normalized contact pressure distributions for the case of compressive line load. In order to include the entire pressure distribution and to give sufficient details in compact forms, different scales have been used for $p(x) > 2$ and for $p(x) < 2$. The contact pressure has a sharp peak at $x = 0$ where the concentrated force is applied. It seems as if the lifted portions of the layer are supported by the rounded reaction around $x = 0$. For a fixed value of the load factor λ the stiffer layer transmits the pressure to the foundation more evenly. The value of the critical load factor λ_{cr} and distance to the location of zero pressure x_{cr} increases as m increases.

In Fig. 11 the size and the location of the separation region for discontinuous contact case under the action of compressive line load are given for $m = 1$. For $\lambda = \lambda_{cr} = 140.39$, $b = c = x_{cr} = 2.94$ h. As $\lambda \to \infty$, which is equivalent to $p_e \to 0$, c/h tends to infinity wheras b/h tends to 1.3. This is the value given in [8] for $p_e = 0$.

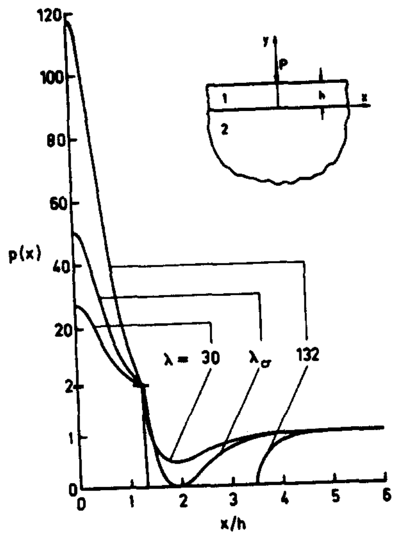

Fig. 8. Normalized contact pressure distributions for $m = 0.1$ (compressive line load case), $\lambda_{cr} = 56.42$, $x_{cr} = 1.86$ h.

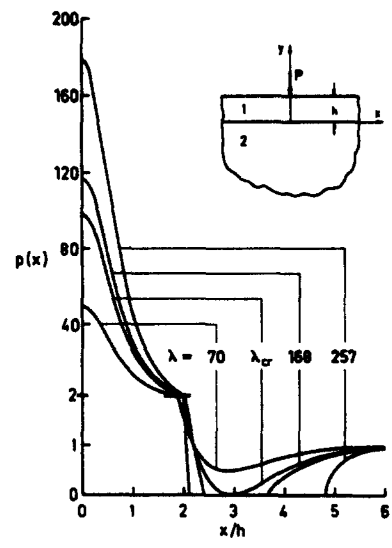

Fig. 9. Normalized contact pressure distributions for $m = 1$ (compressive line load case), $\lambda_{cr} = 140.39$, $x_{cr} = 2.94$ h.

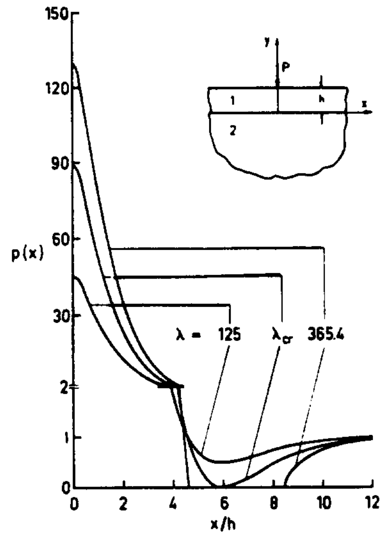

Fig. 10. Normalized contact pressure distributions for $m = 10$ (compressive line load case), $\lambda_{cr} = 250.67$, $x_{cr} = 5.87$ h.

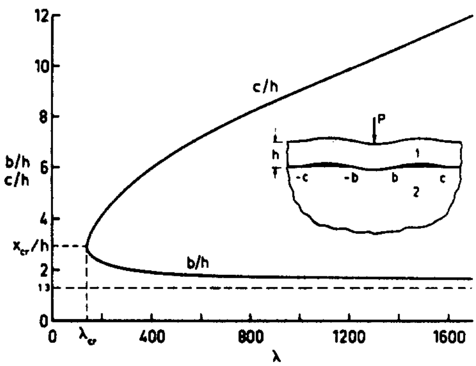

Fig. 11. Location and size of separation region as a function of the load factor λ for $m = 1$ (compressive line load case), $\lambda_{cr} = 140.39$, $x_{cr}/h = 2.94$.

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